Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2016

Marking Scheme

Applied Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. The points P and Q lie on a straight level road.

A car travels along the road in the direction from P to Q. It is initially moving with a uniform speed of 14 m s^{-1}. As it passes P it accelerates uniformly for 8 seconds until it reaches a speed of 30 m s^{-1}.

Then the car decelerates uniformly from a speed of 30 m s^{-1} to a speed of 22 m s^{-1}.
The car travels 52 metres while decelerating.

It now continues at a constant speed of 22 m s^{-1} for 10 seconds and then passes Q.

(a) Draw a speed-time graph of the motion of the car from P to Q.

(b) Find **(i)** the acceleration

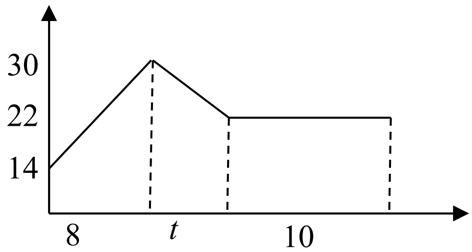
(ii) the deceleration

(iii) $|PQ|$, the distance from P to Q

(iv) the average speed of the car as it travels from P to Q

(v) the time for which the car is moving at or above its average speed.

(a)



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(b) (i) $v = u + at$

$30 = 14 + a(8)$

$a = 2 \text{ m s}^{-2}$ 10

(ii) $v^2 = u^2 + 2as$

$22^2 = 30^2 + 2a(52)$

$a = -4 \text{ m s}^{-2}$ 10

(iii) $s = ut + \frac{1}{2}at^2$

$s_1 = 14(8) + \frac{1}{2}(2)(64) = 176$

$|PQ| = 176 + 52 + 22(10) = 448$ m 10

(iv) $\text{average speed} = \dfrac{448}{20}$

$= 22.4 \text{ m s}^{-1}$ 5

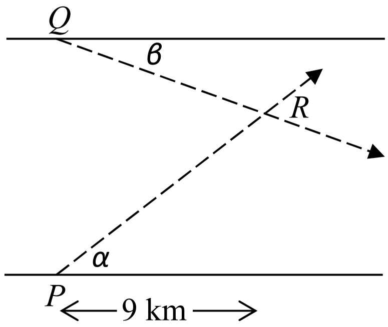
(v) $v = u + at \Rightarrow 30 = 22.4 + 2t_1 \Rightarrow t_1 = 3.8$

$v = u + at \Rightarrow 2.4 = 30 - 4t_2 \Rightarrow t_2 = 1.9$

$t = t_1 + t_2 = 5.7$ s 5

50

2. P is a point on the southern bank of a river.
Q is a point directly opposite P on the northern bank.
Ship A departs from P at a constant speed of 52 km h^{-1} and travels in a direction East $\alpha°$ North, where $\tan\alpha = \frac{12}{5}$.
Ship B departs from Q at a constant speed of 51 km h^{-1} and travels in a direction East $\beta°$ South, where $\tan\beta = \frac{8}{15}$.



Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

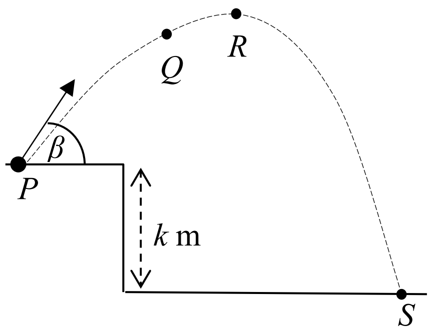
The paths of A and B intersect at point R, which is 9 km downstream from P and Q.

Find **(iv)** the time it takes B to reach R and how much longer it takes A to reach R.

(v) the width of the river, assuming its banks are parallel.

(i)	$\vec{V}_A = 52\cos\alpha\,\vec{i} + 52\sin\alpha\,\vec{j}$	5	
	$= 20\,\vec{i} + 48\,\vec{j}$	5	
(ii)	$\vec{V}_B = 51\cos\beta\,\vec{i} - 51\sin\beta\,\vec{j}$	5	
	$= 45\,\vec{i} - 24\,\vec{j}$	5	
(iii)	$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$	5	
	$= -25\,\vec{i} + 72\,\vec{j}$	5	
(iv)	$t_B = \dfrac{9}{45} = 0.2\text{ h}$	5	
	$t_A = \dfrac{9}{20} = 0.45\text{ h}$		
	$t_A - t_B = 0.25\text{ h}$	5	
(v)	$d = 24 \times 0.2 + 48 \times 0.45$		
	$= 26.4\text{ km}$	10	50

3. A particle is projected from a point P, as shown in the diagram, with an initial speed of 74 m s^{-1} at an angle β to the horizontal, where $\tan\beta = \frac{35}{12}$.



The particle reaches point Q after 4 seconds of motion.
R is the highest point reached by the particle.

Find **(i)** the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of the particle at point Q in terms of $\vec{i}$ and $\vec{j}$

(iii) the displacement of R from P in terms of $\vec{i}$ and $\vec{j}$

(iv) the value of k, given that the particle reaches S after 16 seconds of motion.

(i) $\vec{u} = 74\cos\beta\,\vec{i} + 74\sin\beta\,\vec{j}$

$= 24\,\vec{i} + 70\,\vec{j}$ 10

(ii) $\vec{v}_Q = 24\,\vec{i} + \{70 - 10\times 4\}\vec{j}$

$= 24\,\vec{i} + 30\,\vec{j}$ 10

(iii) $\vec{r}_R = 24t\,\vec{i} + \{70t - \tfrac{1}{2}gt^2\}\vec{j}$ 5

$= 24(7)\,\vec{i} + \{70(7) - 5(7)^2\}\vec{j}$ 5

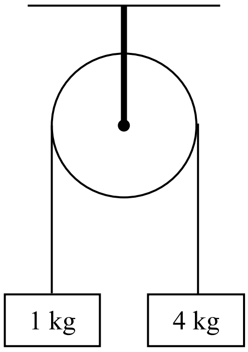
$= 168\,\vec{i} + 245\,\vec{j}$ 5

(iv) $-k = 70(16) - 5(16)^2$ 5

$-k = 1120 - 1280$ 5

$k = 160\text{ m}$ 5 | 50

4. **(a)** Masses of 1 kg and 4 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley.
The system is released from rest.
Find **(i)** the common acceleration of the masses
(ii) the tension in the string.



(i) $4g - T = 4a$ — 5

$T - g = 1a$ — 5

$3g = 5a$

$a = \dfrac{3g}{5} \;=\; 6 \;\; \mathrm{m\,s^{-2}}$ — 5

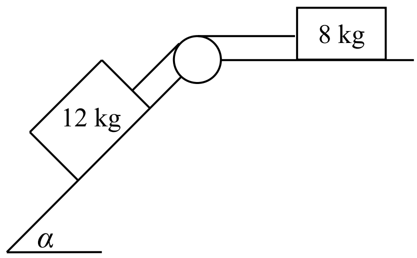
(ii) $T - g = 1a$

$T = g + a$

$= 10 + 6$

$= 16\ \mathrm{N}$ — 5 — 20

(b) Masses of 8 kg and 12 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram.
The 8 kg mass lies on a rough horizontal plane and the coefficient of friction between the 8 kg mass and the plane is $\frac{3}{4}$.
The 12 kg mass lies on a smooth plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{4}{3}$.
The system is released from rest.



(i) Show on separate diagrams the forces acting on each mass.

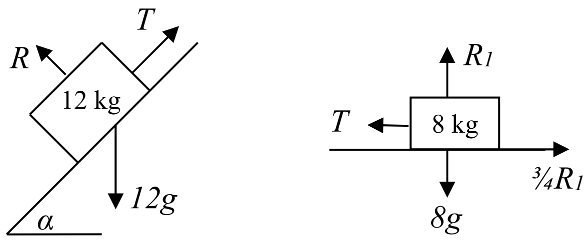
(ii) Find the common acceleration of the masses.

(iii) Find the tension in the string.

(iv) Find the common speed of the masses after two seconds of motion.

(b)

(i)



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(ii)

(ii) $12g\sin\alpha - T = 12a$ 5

$96 - T = 12a$

$T - \frac{3}{4}R_1 = 8a$

$T - \frac{3}{4}(8g) = 8a$

$T - 60 = 8a$

$96 - 60 = 20a$

$a = 1.8\ \text{m s}^{-2}$ 5

(iii) $T - 60 = 8 \times 1.8$

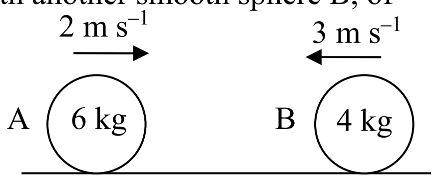
$T = 74.4\ \text{N}$ 5

(iv) $v = u + at$

$= 0 + 1.8 \times 2$

$v = 3.6\ \text{m s}^{-1}$ 5 | 30

5. **(a)** A smooth sphere A, of mass 6 kg, collides directly with another smooth sphere B, of mass 4 kg, on a smooth horizontal table.
Spheres A and B are moving in opposite directions with speeds of 2 m s^{-1} and 3 m s^{-1} respectively.



The coefficient of restitution for the collision is $\frac{2}{5}$.

Find **(i)** the speed of A and the speed of B after the collision
(ii) the loss in kinetic energy due to the collision
(iii) the magnitude of the impulse imparted to A due to the collision.

(b) A ball is fired vertically down with a speed of 2 m s^{-1} from a height of 3 metres onto a smooth horizontal floor. The ball hits the floor and rebounds to a height of 1·8 metres. The coefficient of restitution between the ball and the floor is e.

Find **(i)** the speed of the ball when it hits the floor
(ii) the value of e.

(a) (i)

$$6(2) + 4(-3) = 6v_1 + 4v_2$$ 5

$$0 = 6v_1 + 4v_2$$

$$v_1 - v_2 = -\frac{2}{5}(2+3) = -2$$ 5

$$|v_1| = 0.8 \text{ m s}^{-1} \qquad v_2 = 1.2 \text{ m s}^{-1}$$ 5

(ii)

$$\text{KE}_b = \tfrac{1}{2}(6)(2)^2 + \tfrac{1}{2}(4)(-3)^2 = 30$$

$$\text{KE}_a = \tfrac{1}{2}(6)(0.8)^2 + \tfrac{1}{2}(4)(1.2)^2 = 4.8$$

$$\text{KE}_b - \text{KE}_a = 30 - 4.8 = 25.2 \text{ J}$$ 10

(iii)

$$\text{I} = |(6)(-0.8) - (6)(2)| = 16.8$$ 5

(b) (i)

$$v^2 = 4 + 2\times10\times3$$

$$v = 8 \text{ m s}^{-1}$$ 10

(ii)

$$0 = (e\times8)^2 + 2(-10)(1.8)$$ 5

$$0 = 64e^2 - 36$$

$$e = \frac{3}{4}$$ 5

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6. **(a)** Particles of weight 9 N, 8 N, q N and 2 N are placed at the points $(-4, 3)$, $(8, 6)$, $(p, 5)$ and $(q, -p)$ respectively.

The co-ordinates of the centre of gravity of the system are $(p, 4)$.

Find **(i)** the value of p

(ii) the value of q.

(a)

$$p = \frac{9(-4) + 8(8) + qp + 2q}{19 + q}$$ 5

$$19p - 2q = 28$$

$$4 = \frac{9(3) + 8(6) + q(5) + 2(-p)}{19 + q}$$ 5

$$2p - q = -1$$

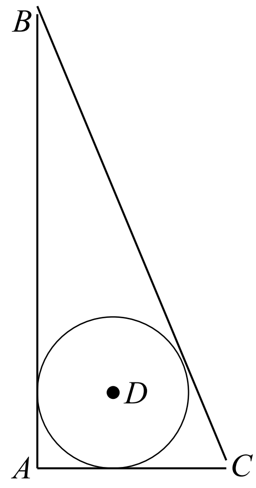
$$p = 2$$ 5

$$q = 5$$ 5 | 20

6 **(b)** A triangular lamina with vertices A, B and C has the portion inside its incircle removed.
D is the centre of the incircle.

The co-ordinates of the points are $A(0, 0)$, $B(0, 108)$, $C(45, 0)$ and $D(18, 18)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.



(b)

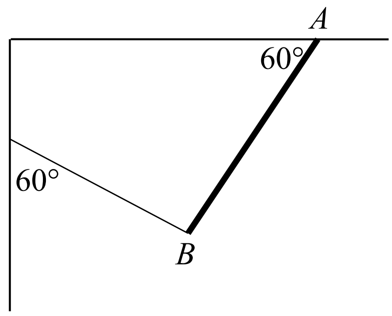
	area :	c.g.	
ABC	$\frac{1}{2}(45)(108) = 2430$	$(15, 36)$	5
Circle	$\pi(18)^2 = 1017.876$	$(18, 18)$	5
lamina	$= 1412.124$	(x, y)	5

$$(1412.124)(x) = 2430(15) - 1017.876(18)$$
$$x = 12.8$$ 10

$$(1412.124)(y) = 2430(36) - 1017.876(18)$$
$$y = 49.0$$ 5

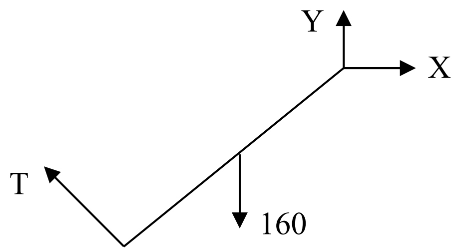
30

7. A uniform rod, AB, of length 4 m and weight 160 N is smoothly hinged at end A to a horizontal ceiling. One end of a light inextensible string is attached to B and the other end of the string is attached to a vertical wall. The rod makes an angle of 60° with the ceiling and the string makes an angle of 60° with the wall, as shown in the diagram. The rod is in equilibrium.



(i) Show on a diagram all the forces acting on the rod AB.
(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
(iii) Write down the equation that arises from taking moments about the point A.
(iv) Find the tension in the string.
(v) Find the magnitude of the reaction at the point A.

(i)



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(ii) $T\cos 30 = X$ 5

$T\sin 30 + Y = 160$ 5

(iii) $T \times 4 = 160 \times 2\cos 60$ 10

(iv) $4T = 160$

$T = 40$ N 10

(v) $X = 40 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}$ 5

$Y = 160 - 40 \times \frac{1}{2} = 140$

$R = \sqrt{\left(20\sqrt{3}\right)^2 + (140)^2}$

$= 144.2$ N 5

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8. **(a)** A particle describes a horizontal circle of radius 1·5 metres with uniform angular velocity ω radians per second. Its speed is 3 m s^{-1} and its mass is 2 kg.

Find **(i)** the value of ω
(ii) the time to complete one revolution
(iii) the centripetal force on the particle.

(i)

$$v = r\omega$$

$$3 = 1.5 \times \omega$$

$$\omega = 2 \text{ rad s}^{-1}$$

10

(ii)

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{2}$$

$$= \pi \text{ s}$$

10

(iii)

$$F = mr\omega^2$$

$$= 2 \times 1.5 \times 4$$

$$= 12 \text{ N}$$

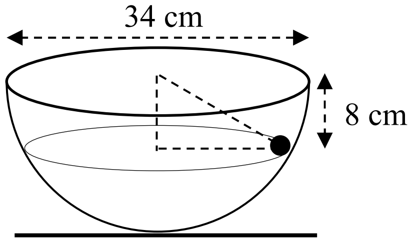
10

30

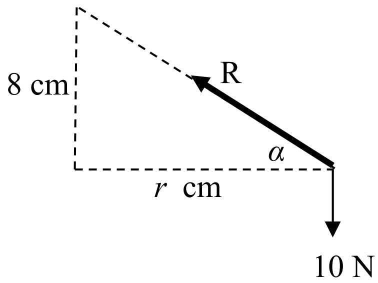
8 (b) A hemispherical bowl of diameter 34 cm is fixed to a horizontal surface.

A smooth particle of mass 1 kg describes a horizontal circle of radius r cm on the smooth inside surface of the bowl.

The plane of the circular motion is 8 cm below the top of the bowl.



Find **(i)** the value of r
(ii) the reaction force between the particle and the surface of the bowl
(iii) the angular velocity of the particle.



(i)	$r^2 + 8^2 = 17^2$	
	$r = 15 \text{ cm}$	5
(ii)	$R\sin\alpha = 10$	
	$R \times \frac{8}{17} = 10$	
	$R = 21.25 \text{ N}$	5
(iii)	$R\cos\alpha = mr\omega^2$	
	$\frac{170}{8} \times \frac{15}{17} = 1 \times 0.15 \times \omega^2$	
	$\omega = 11.18 \text{ rad s}^{-1}$	10

20

9. **(a)** A solid piece of metal has a weight of 23 N.
When it is completely immersed in water, the metal appears to weigh 17 N.

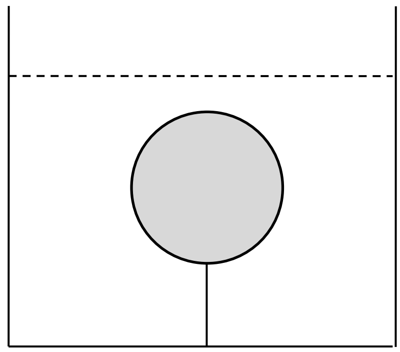
(i) State the principle of Archimedes.
(ii) Find the volume of the metal.
(iii) Find the density of the metal.

[Density of water = 1000 kg m^{-3}]

(b) A solid sphere has a radius of 5 cm.
The density of the sphere is 800 kg m^{-3} and it is completely immersed in a tank of liquid of density 1200 kg m^{-3}.

The sphere is held at rest by a light inextensible, vertical string which is attached to the base of the tank.

Find the tension in the string.



(a)

(i) Principle of Archimedes: 5

(ii)
$$\rho Vg = B$$
$$1000 \times V \times 10 = 6$$
$$V = 0.0006 \text{ m}^3$$
10

(iii)
$$\rho = \frac{m}{V} = \frac{2.3}{0.0006}$$
$$= 3833.3 \text{ kg m}^{-3}$$
5

(b)
$$W = 800\{\tfrac{4}{3}\pi(0.05^3)\}(10)$$
10
$$= \tfrac{4}{3}\pi = 4.189$$
$$B = 1200\{\tfrac{4}{3}\pi(0.05^3)\}(10)$$
10
$$= 2\pi = 6{,}283$$

$$T + W = B$$
$$T = 6.283 - 4.189$$
$$= 2.094 \text{ N}$$
10

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